Chapter 6

State-Space Models

6.1 Introduction

A very general model that seems to subsume a whole class of special cases of interest in much the same way that linear regression does is the state-space model or the dynamic linear model, which was introduced in Kalman (1960) and Kalman and Bucy (1961). Although the model was originally introduced as a method primarily for use in aerospace-related research, it has been applied to modeling data from economics (Harrison and Stevens, 1976; Harvey and Pierse, 1984; Harvey and Todd, 1983; Kitagawa and Gersch 1984, Shumway and Stoffer, 1982), medicine (Jones, 1984) and the soil sciences (Shumway, 1985). An excellent modern treatment of time series analysis based on the state space model is the text by Durbin and Koopman (2001).

Although there are some packages available for R that focus on various aspects of state-space modeling and Kalman filtering, we prefer to write our own code. As a result, the code we have written is long and will most likely be subject to frequent updates. Hence, we have decided to distribute the R code for this chapter on the website for the text.

The state-space model or dynamic linear model (DLM), in its basic form, employs an order one, vector autoregression as the state equation,

$$\boldsymbol{x}_t = \Phi \boldsymbol{x}_{t-1} + \boldsymbol{w}_t, \tag{6.1}$$

where the state equation determines the rule for the generation of the $p \times 1$ state vector $\boldsymbol{x}_t$ from the past $p \times 1$ state $\boldsymbol{x}_{t-1}$, for time points $t = 1, \ldots, n$. We assume the $\boldsymbol{w}_t$ are $p \times 1$ independent and identically distributed, zero-mean normal vectors with covariance matrix Q. In the DLM, we assume the process starts with a normal vector $\boldsymbol{x}_0$ that has mean $\boldsymbol{\mu}_0$ and $p \times p$ covariance matrix Σ_0.

The DLM, however, adds an additional component to the model in assuming we do not observe the state vector $\boldsymbol{x}_t$ directly, but only a linear transformed